Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Mathematics-II

Subject Code: 4SC02MAT1 **Branch: B.Sc. (All)**

Semester: 2 **Date: 09/05/2017** **Time: 02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The solution of the differential equation $(D^2 - 2D + 1)y = 0$ is ………. . (01)

(1) $c_1 e^x + c_2 e^{-x}$ (3) $(c_1 + c_2 x)e^{-x}$
(2) $(c_1 + c_2 x)e^x$ (4) None of these

b) The particular integral of $(D^2 + a^2)y = \sin ax$ is ………….. (01)

(1) $-\frac{x}{2a}\cos ax$ (3) $-\frac{ax}{2}\cos ax$
(2) $\frac{x}{2a}\cos ax$ (4) $\frac{ax}{2}\cos ax$

c) When we put $z = \log x$ in a homogeneous linear differential equation, the value of $x^2 \frac{d^2y}{dx^2}$ is ……….. . (01)

(1) $z^2 \frac{d^2y}{dz^2}$ (3) $\frac{d^2y}{dz^2} - \frac{dy}{dz}$
(2) $z^2 \frac{d^2y}{dz^2} - z\frac{dy}{dz}$ (4) $\frac{d^2y}{dz^2} + \frac{dy}{dz}$

d) The particular integral of the differential equation $(D^2 - 3D + 2)y = e^{5x}$ is … . (01)

(1) e^{5x} (3) $\frac{1}{6}e^{5x}$
(2) $\frac{1}{12}e^{5x}$ (4) $\frac{1}{4}e^{5x}$

e) $\frac{1}{D-m}\,Q$is equal to ……… . (01)

(1) $e^{mx}\int Q\,dx$ (3) $e^{-mx}\int Q\,dx$
(2) $e^{-mx}\int Q\,e^{mx}\,dx$ (4) $e^{mx}\int Q\,e^{-mx}\,dx$

f) The complex conjugate of $\frac{i}{1-i}$ is ………. . (01)

(1) $\frac{-i}{1+i}$ (3) $\frac{i-1}{2}$

(2) $\frac{1-i}{i}$ (4) None of these

g) Real part of $\cosh z$ is ……. . (01)

(1) $\cosh x \cos y$ (3) $\cos h\, x\ \sin y$

(2) $\sinh x \sin y$ (4) $\sinh x \cos y$

h) If $z = \cos\theta + i\sin\theta$, then $\sin n\theta$ = ………… . (01)

(1) $\frac{z^n + z^{-n}}{2}$ (3) $\frac{z^n - z^{-n}}{2i}$

(2) $\frac{z^n - z^{-n}}{2}$ (4) None of these

i) If $x + iy = \sqrt{2} + 3i$, then $x^2 + y$ is ……….. . (01)

(1) 7 (3) 13

(2) 5 (4) $\sqrt{2} + 3$

j) The real part of $(\sin x + i\cos x)^5$ is …………. . (01)

(1) $-\cos 5x$ (3) $\sin 5x$

(2) $-\sin 5x$ (4) $\cos 5x$

k) General equation to the cone which passes through the axes is …….. . (01)

(1) $ax^2 + by^2 + cz^2 = 1$ (3) $fyz + gzx + hxy = 1$

(2) $ax^2 + by^2 + cz^2 = 0$ (4) $fyz + gzx + hxy = 0$

l) The equation of the enveloping cone can be written as: (01)

(1) $S = T^2$ (3) $T = S_1$

(2) $SS_1 = T^2$ (4) None of these

m) Guiding curve of a right circular cylinder is …….. . (01)

(1) ellipse (3) pair of straight lines

(2) circle (4) any closed curve

n) The equation $\frac{x^2}{2} - \frac{y^2}{3} = z$ represents: (01)

(1) cylinder (3) ellipsoid

(2) hyperboloid (4) paraboloid

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the equation of cone whose vertex is (α, β, γ) and base $ax^2 + by^2 = 1, z = 0$. (05)

b) Describe and sketch the conicoid $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{9} = 1$. (05)

c) Find the equation of a cylinder whose generating lines have the direction cosine (l, m, n) and which passes through the circle $x^2 + z^2 = a^2, y = 0$. (04)

Q-3 **Attempt all questions** **(14)**

a) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = 25$, whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$. (05)

b) Prove that the equation $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ (05)

represents a cone whose vertex is $\left(-\frac{7}{6},\frac{1}{3},\frac{5}{6}\right)$.

c) Define: $\log(x+iy)$. Determine $\log(1-i)$. (04)

Q-4 Attempt all questions **(14)**

a) Prove that the n^{th} root of unity are in a geometric progression. Also show that their sum is zero. (05)

b) Separate real and imaginary parts of $\tan(x-iy)$. (05)

c) Solve: $y'' + 16y = x^4 + e^{3x} + \cos 3x$. (04)

Q-5 Attempt all questions **(14)**

a) Solve: $(x^2D^2 - 3xD + 4)y = x^2$, given that $y(1) = 1$ and $y'(1) = 0$. (05)

b) Solve: $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x^2$. (05)

c) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x}$. (04)

Q-6 Attempt all questions **(14)**

a) State and prove De-Moiver's theorem. (05)

b) Prove that $\sinh^{-1}(z) = \ln\left(z + \sqrt{z^2+1}\right)$. (05)

c) Expand $\cos^5\theta$ in a series of cosines of multiples of θ. (04)

Q-7 Attempt all questions **(14)**

a) Solve: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$. (05)

b) Prove that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal. (05)

c) Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$. (04)

Q-8 Attempt all questions **(14)**

a) Solve the simultaneous equations $\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dx} - 2x - \cos t = 0$ given that $x = 0$ and $y = 1$ when $t = 0$. (05)

b) Solve: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$. (05)

c) Identify the surface given by $9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$. (04)